DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

120. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics in Boys' High School, Reading, Pa.

How many balls 1 inch in diameter can be put in a cubical box 1 foot in the *clear* each way, putting in the maximum number? [From Greenleaf's *Treatise on Algebra.*]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and the PROPOSER.

First, put in two layers of 144 balls each, then fourteen layers of 121 and 144 balls in alternate layers.

This gives nine layers of 144 each=1296
seven layers of 121 each= 847

Total=2143

Every four balls in 144 layer makes with each ball in a 121 layer a square pyramid, base 1 inch, and slant edge 1 inch.

$\therefore$ Altitude$=\frac{1}{2}\sqrt{2}=.7071$ inches.

$.7071\times14=9.8994$ inches. $9.8994+2=11.8994$ inches.

$12-11.8994=.1006$ inch to spare.

Also solved by *CHARLES C. CROSS.*

121. Proposed by PAUL ROULET, A. M., Professor of Mathematics, Fairmount College, Wichita, Kas.

Three men, named Adams, Morris, and Stoughton, with their sons, Edward, Nathan, and Walter, have each a piece of land in the form of a square. Mr. Adams' piece is 23 rods longer on each side than Nathan's, and Mr. Stoughton's piece is 11 rods longer on each side than Edward's. Each father has 63 square rods of land more than his son. Which of these persons is father and son, respectively?

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let the initial letter of person's name represent the length of the respective person's square piece of land.

Then $63+E^2=\square$, $63+N^2=\square$, and $63+W^2=\square$.

We are now to find which of these squares equals, respectively, A^2, M^2, and S^2.

The basis of work is the identity,

$$mn+\left(\frac{m-n}{2}\right)^2=\left(\frac{m+n}{2}\right)^2,$$

where $mn=63=9\times7=21\times3=63\times1$.

From these sets of factors of 63, we obtain

$$63+1^2=8^2,\ 63+9^2=12^2,\ \text{and}\ 63+31^2=32^2.$$

Hence 1^2, 9^2, and 31^2 are the pieces of land belonging to the sons, and 8^2, 12^2, and 32^2 those belonging to their respective fathers.

But, from conditions of problem, we have $A-23=N$, and $S-11=E$.

Whence, it is evident that A must equal 32, and S must equal 12, from which we find $N=9$, and $E=1$.

Therefore, of the two remaining values, $M=3$, and $W=31$.

Now, since each father has 63 square rods of land more than his son, the man having 8^2 is the father of the son having 1^2; the man having 12^2 is the father of the son having 9^2; and so on.

$\therefore$ Mr. Morris is Edward's father; Mr. Stoughton is Nathan's father; and Mr. Adams is Walter's father.

A similar problem, said to be published in several newspapers of recent issue, is as follows: "Three Dutchmen, Hans, Klaus, and Hendricks, went to market to buy hogs, and took their wives with them. The names of the wives were Gertrude, Anna, and Katrine; but it was not known which was the wife of each man. They each, men and wives, bought as many hogs as each paid shillings, respectively, for each hog; and each man spent three guineas more than his wife. Hendricks bought 23 hogs more than Gertrude, and Klaus bought 11 more than Katrine. What was the name of each man's wife?"

A solution, similar to the above, gives Katrine as Hans's wife, Gertrude as Klaus's wife, and Anna as Hendricks's wife.

II. Solution by W. H. CARTER, Vice President and Professor of Mathematics, Centenary College, Jackson, La.

It is shown in geometry that the difference between two square areas is equal to a rectangle whose base is the sum of the sides of the squares and whose altitude is the difference between the sides.

63 square rods is the area of this rectangle. Then 63 is the product of two factors one of which is the sum and the other the difference of the sides of the pieces of father and son.

$63=63\times1$ or 21×3 or 9×7.

The side of the greater square, or the father's, is obtained by adding the sum and difference and dividing by two, and of the smaller, or son's, by subtracting the difference from the sum and dividing by two.

This process applied to each of the above pairs of factors gives for the sides of the pieces of father and son, respectively, 32 and 31, 12 and 9, 8 and 1. Since 31 and 9 differ by 23, 32 is the side of Adams's piece, and 9 is the side of Nathan's piece. Also 12 and 1 differ by 11. Therefore 12 is the side of Stoughton's piece, and 1 is the side of Edward's. This leaves 8 for the side of Morris's piece, and 31 for the side of Walter's.

Therefore the boys' names are Edward Morris, Nathan Stoughton, and Walter Adams.

III. Solution by B. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, O.

I. 1. Nathan is not the son of Mr. Adams, since the former has more than 63 square rods of land in excess of the latter.

Therefore either Edward or Walter is the son of Mr. Adams.

2. Edward is not the son of Mr. Stoughton, since the former has more than 63 square rods of land in excess of the latter.

Therefore Nathan or Walter is the son of Mr. Stoughton.

3. Walter is not the son of Mr. Morris, for then would Mr. Adams and Mr. Stoughton each have more land than the other, which is absurd. For then would Edward be the son of Mr. Adams, and Nathan of Mr. Stoughton; but Mr. Adams and Mr. Stoughton have more than 63 square rods, respectively, than Nathan and Edward, and, hence, more than each other.

Therefore either Nathan is the son of Mr. Morris, Walter of Mr. Stoughton, and Edward of Mr. Adams, or Edward is the son of Mr. Morris, Walter of Mr. Adams, and Nathan of Mr. Stoughton.

5. This indefinite solution suggests the idea that the proposer intended as a condition what is neither expressly stated nor necessarily implied in the problem, viz., that all the numbers are integral.

The second solution is on this assumption.

II. 1. Let x stand for the side, in rods, of any square, and $x+y$ for the side of a square whose length is y rods longer. Then $2xy+y^2=63$, whence $x=(63/2y)-\frac{1}{2}y$.

Now, the only positive integral values of y which will make x a positive integer are 1, 3, and 7, the values of x corresponding to which are, respectively, 31, 9, and 1. Hence, the pairs of squares differing by 63 square rods are as follows: (1) Sides, 32 rods and 31 rods; (2) 12 rods and 9 rods; (3) 8 rods and 1 rod.

2. Now, applying the first condition of the problem, it is evident that Mr. Adams owns the largest square, and that his son is Walter, that Nathan is the son of Mr. Stoughton, and Edward of Mr. Morris.

IV. Solution by SYLVESTER ROBINS, North Branch Depot, N. J.

$63=63\times1=21\times3=9\times7=32^2-31^2=12^2-9^2=8^2-1^2$.

$32-9=23$.

32^2 belongs to father Adams. 31^2 belongs to Mr. Adams's son.

9^2 belongs to Nathan. 12^2 to Nathan's fathər.

$12-1=11$.

The 12^2 belongs to father Stoughton. 1^2 belongs to Edward. 9^2 belongs to Nathan Stoughton.

The 8^2 belongs to father Morris. 1^2 to Edward Morris, and the 31^2 to Walter Adams.

Also solved by *COOPER D. SCHMITT*, *J. SCHEFFER*, and *G. B. M. ZERR*.

NOTE. Professor Scheffer sent in solutions of problems 118 and 119, and Professor Ellwood of problem 119. In the published solutions of No. 119, the following errors occur in the statement : For $1.\frac{9}{.100}$, read $1.\frac{9}{.001}$; line 8 from the bottom, for "\$9001, the selling price," read "\$9001, the cost price;" at top of page 270, for $1+\frac{9}{.009}$ read $1+\frac{9}{.001}$

ALGEBRA.

97. Proposed by F. M. SHIELDS, Coopwood, Miss.

A farmer had 2080 pounds of grain at the depot, and gave a wagoner .75 cents per 100 pounds to haul it, paying him in the *same* grain at the following prices, viz.: 3-10 of the hauling bill was paid in corn at .58 cents per bushel of 56 pounds; 3-5 was paid in wheat at 1.55 cents per bushel of 60 pounds, and the balance of the bill was paid in oats at .36 cents per bushel of 32 pounds, the wagoner not charging for hauling his own grain. The load being delivered, how many bushels of each kind of grain did the wagoner get, and how many bushels of each kind did the farmer have left after paying the wagoner?

I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and C. B. GOULD, Colorado College, Colorado Springs, Col.

Let x=the hauling bill.

$x\div.75=\frac{4}{3}x$=number of hundredweight hauled.

$\therefore$ $400x/3$=number of pounds hauled.

$(\frac{3}{10}x\div.58)56=\frac{3}{10}x\times\frac{50}{29}\times\frac{56}{1}=840x/29$, pounds corn received by wagoner.

$(\frac{6}{10}x\div1.55)60=\frac{6}{10}x\times\frac{20}{31}\times\frac{60}{1}=720x/31$, pounds wheat received by wagoner.

$(\frac{1}{10}x\div.36)32=\frac{1}{10}x\times\frac{25}{9}\times\frac{32}{1}=80x/9$, pounds oats received by wagoner.

$\therefore$ $2080-400x/3=840x/29+720x/31+80x/9$.

$\therefore$ $x=\$10.698843$, hauling bill.

$3x/5.80=5.5338843$ bushels of corn wagoner received.

$6x/15.50=4.1414877$ bushels of wheat wagoner received.

$x/3.60=2.9719008$ bushels of oats wagoner received.

$400x/3=1426.5124$ pounds hauled for farmer.

$\frac{3}{10}$ of $400x/3=40x$, $\frac{6}{10}$ of $400x/3=80x$, $\frac{1}{10}$ of $400x/3=40x/3$.

$40x\div56=7.6420307$ bushels of corn farmer had left.

$80x\div60=14.2651240$ bushels of wheat farmer had left.

$40x/3\div32=4.45785125$ bushels oats farmer had left.

II. Solution by J. D. CRAIG, Frankfort, N. J., and P. S. BERG, A. M., Principal of Schools, Larimore, N. D.

Let $40x$=number of pounds of grain farmer retained.

Then $\frac{3}{4}\times40x=30x$=cents paid for carting.

$\frac{3}{10}$ of $30x=9x$=cents paid for carting corn.

$\frac{3}{5}$ of $30x=18x$=cents paid for carting wheat.

$\frac{1}{10}$ of $30x=3x$=cents paid for carting oats.

$9x\div\frac{3}{4}=12x$=pounds of corn farmer retained.

$18x\div\frac{3}{4}=24x$=pounds of wheat farmer retained.